

 $a_{n+1} = a_n + (n+1)$ 

then P(n) is true for every NEN.

Example. For each nerv,  $P(n): \sum_{k=1}^{\infty} G = n(n+1)$ . P(1) is true n=1: On one hand, 1+2 $\sum_{k=1}^{1} k = 1$ 

On the other hand  $\frac{n(n+1)}{2} = \frac{1(1+1)}{2} = 1$ 

P(1) is true I. H ---- P(n+1) 1 p P(n) is true, then P(nti) is true Now, assume that P(n) is true (I.H). We need to prove that P(n+1) is troe.  $P(n_{ti}): \overset{n_{ti}}{\underset{k=1}{\overset{}}} k = (n_{ti})(n_{ti})$ 2

 $\sum_{k=1}^{n+1} K = \sum_{k=1}^{n} k + (n+1) = \frac{1.4}{2}$ 

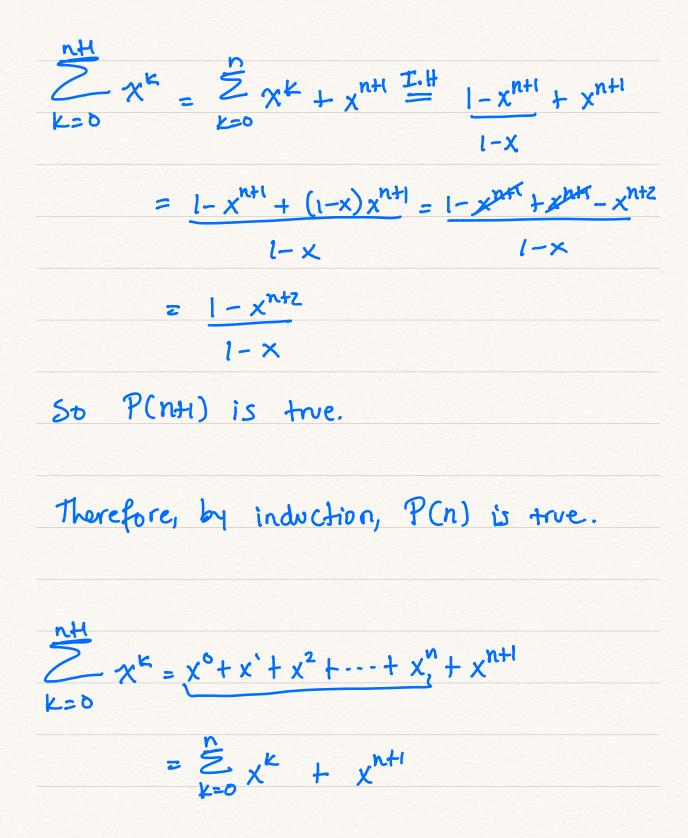
 $= n(n_{H}) + 2(n_{H}) = (n_{H})(n_{H})$ 2

Therefore, by induction, P(n) is true for all nelN.

Example. If 
$$x \neq 1$$
, then  

$$P(n): \sum_{k=0}^{n} x^{k} = \underbrace{1-x^{nH}}_{1-x} \quad \forall n \in \mathbb{N}.$$

$$1-x \qquad \mathbb{N} = \frac{3}{1}, 2, 3, ..., \frac{3}{2}$$
1. On one hand,  $\sum_{k=0}^{\pm} x^{k} = x^{0} + x = 1 + x$   
On the other hand,  $\underbrace{1-x^{2}}_{1-x} = \underbrace{(1-x)(1+x)}_{1-x} = 1 + x$   
So  $P(1)$  is true.  
2. Assume that  $P(n)$  is true.  
I need to show that  $P(nti)$  is true.  
 $P(nti): \sum_{k=0}^{n+1} x^{k} = \underbrace{1-x^{n+2}}_{1-x}$ 



Example For all new 

 $a^{n}-b^{n}=(a-b)\sum_{k=1}^{n}a^{n-k}b^{k-1}$ 

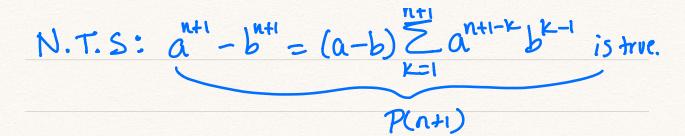
 $a^{2} - b^{2} = (a - b)(a + b)$  $a^{3} - b^{3} = (a - b) \stackrel{2}{\leq} a^{3-k} b^{k-1} = (a - b)(a^{2} + ab + b^{2})$ 

P(1)? (On one hand,  $a^{1}-b^{1}$ On the hand,  $(a-b) \stackrel{1}{\geq} a^{1-k} b^{k-1} = (a-b)$ 

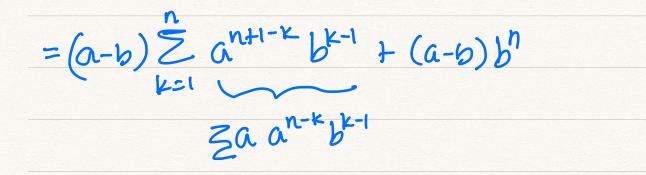
So P(1) is true.

Induction step

I.H:  $a^{n} - b^{n} = (a-b) \sum_{k=1}^{m} a^{n-k} b^{k-1}$  is force

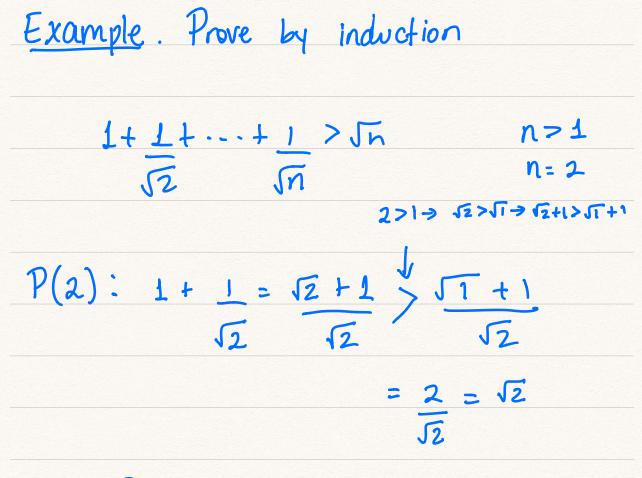


 $(a-b) \stackrel{n+1}{\geq} a^{n+1-k} b^{k-1} =$  $(a-b)\left[\sum_{k=1}^{k}a^{n+1-k}b^{k-1}+a^{n+1-(n+1)}b^{(n+1)-1}\right]$ 



 $= a(a-b) \sum_{k=1}^{n} a^{n-k} b^{k-1} + (a-b) b^{n}$ Z.H: an-bn

 $= \alpha (a^{n} - b^{n}) + (a - b)b^{n} = a^{n+1} - gb^{n} + gb^{n} - b^{n+1}$ = (n+1 - bn+1



## So P(2) is true.

Induction Step! IH: 1+ 1+...+ 1 > Jn is true JZ JN NTS: 1+1+...+1+1>Jn+1 is true JZ JN Jn+1

